# Homework 8

## Solution

P 8.3 **[a]** $i_R(0) = \dfrac{15}{200} = 75\text{mA}$

$i_L(0) = -45\text{mA}$

$i_C(0) = -i_L(0) - i_R(0) = 45 - 75 = -30\,\text{mA}$

**[b]** $\alpha = \dfrac{1}{2RC} = \dfrac{1}{2(200)(0.2 \times 10^{-6})} = 12{,}500$

$\omega_o^2 = \dfrac{1}{LC} = \dfrac{1}{(50 \times 10^{-3})(0.2 \times 10^{-6})} = 10^8$

$s_{1,2} = -12{,}500 \pm \sqrt{1.5625 \times 10^8 - 10^8} = -12{,}500 \pm 7500$

$s_1 = -5000$ rad/s; $\quad s_2 = -20{,}000$ rad/s

$v = A_1 e^{-5000t} + A_2 e^{-20{,}000t}$

$v(0) = A_1 + A_2 = 15$

$\dfrac{dv}{dt}(0) = -5000A_1 - 20{,}000A_2 = \dfrac{-30 \times 10^{-3}}{0.2 \times 10^{-6}} = -15 \times 10^4 \text{V/s}$

Solving, $\quad A_1 = 10; \quad A_2 = 5$

$v = 10e^{-5000t} + 5e^{-20{,}000t}\,\text{V}, \qquad t \geq 0$

**[c]** $i_C = C\dfrac{dv}{dt}$

$= 0.2 \times 10^{-6}[-50{,}000e^{-5000t} - 100{,}000e^{-20{,}000t}]$

$= -10e^{-5000t} - 20e^{-20{,}000t}\,\text{mA}$

$i_R = 50e^{-5000t} + 25e^{-20{,}000t}\,\text{mA}$

$i_L = -i_C - i_R = -40e^{-5000t} - 5e^{-20{,}000t}\,\text{mA}, \quad t \geq 0$

P 8.10 **[a]** $\alpha = \dfrac{1}{2RC} = 0.5$ rad/s

$$\omega_o^2 = \frac{1}{LC} = 25.25$$

$$\omega_d = \sqrt{25.25 - (0.5)^2} = 5 \text{ rad/s}$$

$$\therefore \quad v = B_1 e^{-t/2}\cos 5t + B_2 e^{-t/2}\sin 5t$$

$$v(0) = B_1 = 0; \qquad v = B_2 e^{-t/2}\sin 5t$$

$$i_R(0^+) = 0\,\text{A}; \qquad i_C(0^+) = 4\,\text{A}; \qquad \frac{dv}{dt}(0^+) = \frac{4}{0.08} = 50\ \text{V/s}$$

$$50 = -\alpha B_1 + \omega_d B_2 = -0.5(0) + 5B_2$$

$$\therefore \quad B_2 = 10$$

$$\therefore \quad v = 10e^{-t/2}\sin 5t\,\text{V}, \qquad t \geq 0$$

**[b]** $\dfrac{dv}{dt} = -5e^{-t/2}\sin 5t + 10e^{-t/2}(5\cos 5t)$

$$\frac{dv}{dt} = 0 \quad \text{when} \quad 10\cos 5t = \sin 5t \quad \text{or} \quad \tan 5t = 10$$

$$\therefore \quad 5t_1 = 1.47, \qquad t_1 = 294.23\,\text{ms}$$

$$5t_2 = 1.47 + \pi, \qquad t_2 = 922.54\,\text{ms}$$

$$5t_3 = 1.47 + 2\pi, \qquad t_3 = 1550.86\,\text{ms}$$

**[c]** $t_3 - t_1 = 1256.6\,\text{ms}; \qquad T_d = \dfrac{2\pi}{\omega_d} = \dfrac{2\pi}{5} = 1256.6\,\text{ms}$

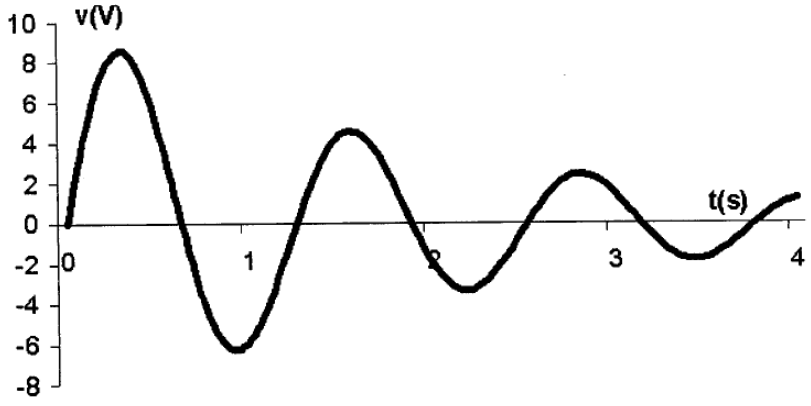
**[d]** $t_2 - t_1 = 628.3\,\text{ms}; \qquad \dfrac{T_d}{2} = \dfrac{1256.6}{2} = 628.3\,\text{ms}$

**[e]** $v(t_1) = 10e^{-(0.147115)}\sin 5(0.29423) = 8.59\,\text{V}$

$$v(t_2) = 10e^{-(0.46127)}\sin 5(0.92254) = -6.27\,\text{V}$$

$$v(t_3) = 10e^{-(0.77543)}\sin 5(1.55086) = 4.58\,\text{V}$$

**[f]**



P 8.29 $\alpha = \dfrac{1}{2RC} = \dfrac{1}{2(400)(1.25 \times 10^{-6})} = 1000$

$\omega_o^2 = \dfrac{1}{LC} = \dfrac{1}{(1.25 \times 10^{-6})(1.25)} = 64 \times 10^4$

$s_{1,2} = -1000 \pm \sqrt{1000^2 - 64 \times 10^4} = -1000 \pm 600$ rad/s

$s_1 = -400$ rad/s; $\qquad s_2 = -1600$ rad/s

$v_o(\infty) = 0 = V_f$

$\therefore \quad v_o = A_1' e^{-400t} + A_2' e^{-1600t}$

$v_o(0) = 12 = A_1' + A_2'$

Note: $\qquad i_C(0^+) = 0$

$\therefore \quad \dfrac{dv_o}{dt}(0) = 0 = -400A_1' - 1600A_2'$

Solving, $\qquad A_1' = 16\,\text{V}, \qquad A_2' = -4\,\text{V}$

$v_o(t) = 16e^{-400t} - 4e^{-1600t}\,\text{V}, \qquad t > 0$

P 8.42 **[a]** $t < 0$:

$$i_o = \frac{80}{800} = 100\,\text{mA}; \qquad v_o = 500 i_o = (500)(0.01) = 50\,\text{V}$$

$t > 0$:

$$\alpha = \frac{R}{2L} = \frac{500}{2(2.5 \times 10^{-3})} = 10^5 \text{ rad/s}$$

$$\omega_o^2 = \frac{1}{LC} = \frac{1}{(2.5 \times 10^{-3})(40 \times 10^{-9})} = 100 \times 10^8$$

$$\alpha^2 = \omega_o^2 \quad \therefore \quad \text{critically damped}$$

$$\therefore \quad i_o(t) = D_1 t e^{-10^5 t} + D_2 e^{-10^5 t}$$

$$i_o(0) = D_2 = 100\,\text{mA}$$

$$\frac{di_o}{dt}(0) = -\alpha D_2 + D_1 = 0$$

$$\therefore \qquad D_1 = 10^5(100 \times 10^{-3}) = 10{,}000$$

$$i_o(t) = 10{,}000 t e^{-10^5 t} + 0.1 e^{-10^5 t}\,\text{A}, \qquad t \geq 0$$

**[b]** $v_o(t) = D_3 t e^{-10^5 t} + D_4 e^{-10^5 t}$

$$v_o(0) = D_4 = 50$$

$$C\frac{dv_o}{dt}(0) = -0.1$$

$$\frac{dv_o}{dt}(0) = \frac{-0.1}{40 \times 10^{-9}} = -25 \times 10^5\,\text{V/s} = -\alpha D_4 + D_3$$

$$\therefore \qquad D_3 = 10^5(50) - 25 \times 10^5 = 25 \times 10^5$$

$$v_o(t) = 25 \times 10^5 t e^{-10^5 t} + 50 e^{-10^5 t}\,\text{V}, \quad t \geq 0$$